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SUPersonic NOZZLE DESIGN

By  
Dr. Stuart L. Brodsky

22 SEPTEMBER 1970



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UNITED STATES NAVAL ORDNANCE LABORATORY, WHITE OAK, MARYLAND

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**ABSTRACT:** This paper is concerned with a method of finite differences for determining two-dimensional and axisymmetric supersonic nozzle contours. The approach taken is to specify a Mach number or velocity array along the entire centerline of the nozzle and then to integrate the equations numerically to obtain the desired nozzle shape. In spite of the fact that the original problem is not "well posed" in the subsonic region, reasonable results were found provided the Mach number gradient was not too steep in a neighborhood of the sonic line.

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**Supersonic Nozzle Design**

This report investigates a method of finite differences for determining two-dimensional and axisymmetric supersonic nozzle contours. This work was supported by the Independent Exploratory Development Program (IED). The author wishes to thank Dr. Richard Roberts for introducing him to the problem and to Mrs. Glyndon Dabbs, who typed the manuscript and aided in obtaining the figures. The assistance of Dr. Jay Solomon in many of the fluid dynamics questions which arose is also gratefully acknowledged.

GEORGE G. BALL  
Captain, USN  
Commander



E. K. RITTER  
By direction

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## CHAPTER I

## INTRODUCTION

The study of two dimensional and axisymmetric converging-diverging nozzles has received a great deal of attention by both engineers and mathematicians. Its uses in wind tunnels and rockets make it of practical importance whereas its mixed mathematical nature give it theoretical interest.

A converging-diverging nozzle (see Figure 1) consists of a converging section where the flow is subsonic and the mathematical equations are elliptic, a diverging section where the flow is supersonic and the equations are hyperbolic, and an intermediate transonic section, called the nozzle throat, where the flow passes from subsonic to supersonic and both elliptic and hyperbolic behavior are present. In wind tunnel applications there is a fourth section, the test section, where the flow is uniform.

There are basically two approaches taken when investigating nozzles - direct and indirect. In the direct method a nozzle contour is prescribed and one seeks to determine the flow field inside. In the indirect method the flow along the axis of symmetry, called the centerline, is prescribed and the nozzle giving rise to this flow is sought. In the design of nozzles the natural method to use is the indirect method and thus will be what we shall consider here. For information on direct methods the reader is referred to [3] and [9].

The two primary indirect methods in use are the series methods and the method of characteristics. In the series methods, originated by Friedrichs [4], (see also [5]), one assumes that the variables of interest can be expanded in series involving the stream variables. These are then substituted into the equations to determine the values of the coefficients in the expansions. The series are then truncated and the remaining finite number of terms is used to compute approximations to the solutions. The method of characteristics ([10], [11])

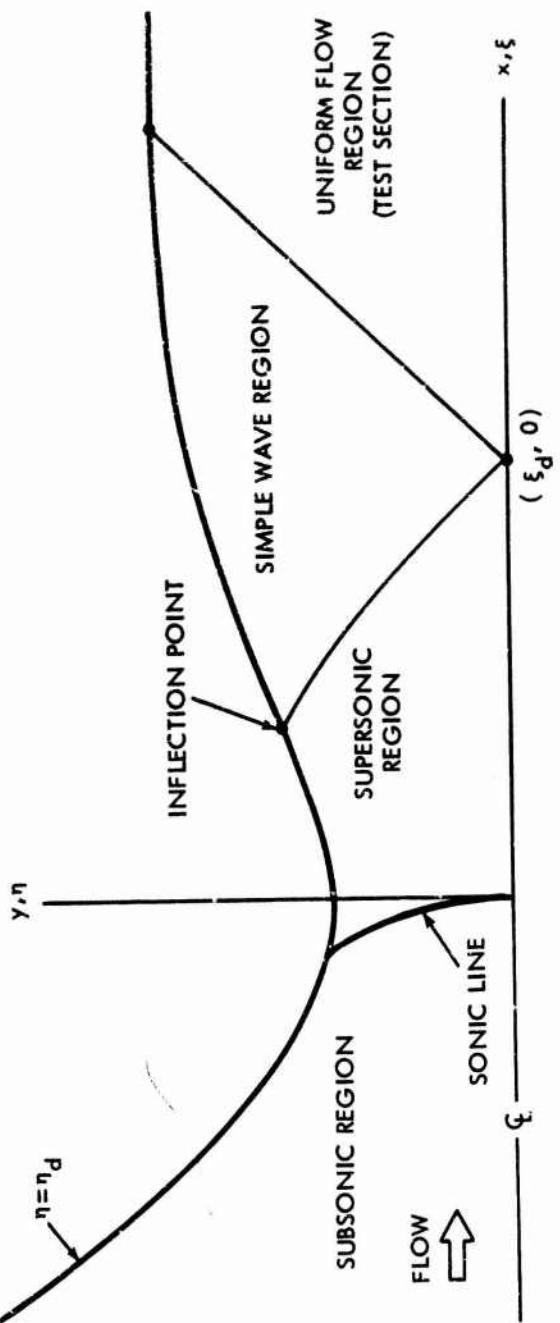


FIG. 1 GEOMETRY OF TWO-DIMENSIONAL AND AXISYMMETRIC NOZZLES

utilizes a change of variables to characteristic coordinates. This transformation makes the two independent variables in the original equation additional dependent variables. One is thus led to a system of four partial differential equations in four unknowns. The simplification occurs in that each of the four new equations contains differentiations with respect to only one of the new independent variables. This enables one to determine the characteristics and the flow in the nozzle. This method, of course, is restricted to the supersonic region where the equations are hyperbolic and the characteristics are real.

The great difficulty in the series methods is the lack of mathematical justification. Little, if any, has been accomplished in analyzing the size of the terms which are dropped. The method of characteristics on the other hand can be used only in the supersonic region. To get around this difficulty different methods are used in each of the different sections and the solutions obtained are then "patched" together with a "French curve." Thus, for example, one might use the method of characteristics in the supersonic region and patch the solution so obtained to the solution obtained in the subsonic and transonic regions by the series method or by a quasi-one-dimensional flow approximation. However, in many applications such as very high Mach number wind tunnels where heat transfer in the throat of the tunnel is sizable and in short nozzles where the two dimensional effects cannot be ignored, the method of patching leaves much to be desired. It is thus desirable to investigate further methods for the determination of nozzle contours for prescribed centerline conditions.

In this paper we consider an indirect method which is usable from the subsonic region to the supersonic test section of constant state. The procedure is to prescribe a Mach number or velocity distribution along the axis of symmetry and then to integrate the governing equations of motion using finite differences. We do not claim to present here a mathematically rigorous technique but rather

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offer another method of calculation which has advantages and disadvantages when compared with other methods. In fact it will be shown in the next chapter when we derive the equations of motion that this procedure is unstable in the subsonic region (as one would expect). We shall say more about this in Chapter IV where we discuss our results and conclusions. The primary advantage of this type of method is that it provides a uniform scheme from the subsonic region to the test section, thus eliminating the need for multiple patching. The disadvantages will be discussed in Chapter IV.

## CHAPTER II

## DEVELOPMENT AND STABILITY OF THE DIFFERENTIAL EQUATIONS

We suppose in the following that we are dealing with a steady, irrotational, perfect gas with constant specific heats. We consider both two-dimensional and axisymmetric flows. For such flows the equation of continuity takes the form

$$(2.1) \quad v \cdot \left( \frac{\rho q}{\rho_0} \right) = \frac{1}{\rho_0} \left[ \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{k \rho v}{y} \right] = 0$$

where  $k = 0$  for two-dimensional flow and  $k = 1$  for axisymmetric flow. The equation of state is

$$(2.2) \quad \frac{p}{p_0} = \left( \frac{\rho}{\rho_0} \right)^\gamma$$

and Bernoulli's equation can be written

$$(2.3) \quad \left( \frac{a_0}{a} \right)^2 = 1 + \frac{\gamma-1}{2} M^2$$

An explanation of the notation can be found on page 18, and a derivation of the above equations from the basic equations of fluid dynamics can be found in any standard book on gasdynamics (see e.g., [12]).

Equation (2.1) implies the existence of a stream function  $\psi$  such that

$$(2.4) \quad \frac{\partial \psi}{\partial y} = \frac{\rho}{\rho_0} u y^k, \quad \frac{\partial \psi}{\partial x} = - \frac{\rho}{\rho_0} v y^k$$

and the assumption of irrotationality implies the existence of a velocity potential  $\phi$  such that

$$\vec{q} = \nabla\phi$$

or

$$(2.5) \quad u = \frac{\partial\phi}{\partial x}, \quad v = \frac{\partial\phi}{\partial y}$$

Substituting (2.5) into (2.4) we obtain the equations

$$(2.6) \quad \frac{\partial\psi}{\partial y} = -\frac{\rho}{\rho_0} y^k \frac{\partial\phi}{\partial x}, \quad \frac{\partial\psi}{\partial x} = -\frac{\rho}{\rho_0} y^k \frac{\partial\phi}{\partial y}$$

Finally, from (2.2) and (2.3) it follows that

$$(2.7) \quad \frac{\rho_0}{\rho} = (1 + \frac{\gamma-1}{2} M^2)^{\frac{1}{\gamma-1}}$$

We next introduce a set of dimensionless variables according to the following definitions:

$$(2.8) \quad x' = \frac{x}{L}, \quad y' = \frac{y}{L}, \quad \phi' = \frac{\phi}{a_0 L}, \quad \psi' = \frac{\psi}{a_0 L^{k+1}}, \quad \rho' = \frac{\rho}{\rho_0}$$

where  $L$  is some standard length. The previous equations then become (with the primes dropped for convenience)

$$(2.9) \quad \left\{ \begin{array}{l} \frac{\partial \psi}{\partial y} = \rho y^k \frac{\partial \phi}{\partial x}, \quad \frac{\partial \psi}{\partial x} = -\rho y^k \frac{\partial \phi}{\partial y} \\ \rho = \left(1 + \frac{\gamma-1}{2} M^2\right)^{-\frac{1}{\gamma-1}} \end{array} \right.$$

Interchanging the role of dependent and independent variables (which is permissible since the Jacobian of the transformation,

$$(2.10) \quad J\left(\frac{\phi, \psi}{x, y}\right) = \rho y^k (u^2 + v^2),$$

is nonzero off the axis) and introducing new variables  $\xi$  and  $\eta$  defined by

$$(2.11) \quad \phi = \int_0^\xi \bar{q}(\tau) d\tau, \quad \psi = \frac{1}{k+1} \rho^* q^* \eta^{k+1}$$

(2.9) becomes

$$(2.12) \quad \left\{ \begin{array}{l} \frac{\partial y}{\partial \eta} = \frac{\rho^* q^*}{\rho \bar{q}} \left(\frac{\eta}{y}\right)^k \frac{\partial x}{\partial \xi}, \quad \frac{\partial x}{\partial \eta} = -\frac{\rho^* q^*}{\rho \bar{q}} \left(\frac{\eta}{y}\right)^k \frac{\partial y}{\partial \xi} \\ \rho = \left(1 - \frac{\gamma-1}{2} \frac{\bar{q}^2(\xi)}{x_\xi^2 + y_\xi^2}\right)^{\frac{1}{\gamma-1}} \end{array} \right.$$

where  $\bar{q}$  denotes the velocity magnitude along the axis of symmetry and  $\rho^*$  and  $q^*$  are the sonic density and velocity magnitude which are fixed for given stagnation conditions.

It is the system (2.12) which we integrate to find the nozzle contour. We view the problem as an initial value problem with information imposed for

$\eta = 0$ , the nozzle centerline, and wish to determine the solution for positive  $\eta$ . We require along  $\eta = 0$  that  $y = 0$  and  $x = \xi$ . We furthermore prescribe a Mach number distribution or equivalently a velocity distribution along this line. This information determines  $x$ ,  $y$ , and  $\rho$  and the derivatives of  $x$  and  $y$  on the centerline using (2.12). This is straightforward except perhaps the determination of  $\frac{\partial y}{\partial \eta}$  in the axisymmetric case,  $k = 1$ , where we must remove the indeterminacy of  $\frac{\eta}{y}$  along the centerline. This is done by replacing  $\frac{\eta}{y}$  by  $\frac{\partial y}{\partial \eta}$  to obtain the relationship

$$\frac{\partial y}{\partial \eta} = \left( \frac{\rho^* q^*}{pq} \right)^{1/2}.$$

As the lines  $\eta = \text{constant}$  correspond to streamlines, any positive  $\eta$  serves as a nozzle boundary. The particular  $\eta$  chosen to terminate the integration is determined by various design or convenience considerations. The numerical procedure used to perform this integration is discussed in the next section.

We conclude this section with an analysis of stability as suggested by Von Neumann and Richtmyer [13]. We suppose that a perturbation  $\delta x$ ,  $\delta y$ ,  $\delta \rho$  is introduced and attempt to see the effect of this perturbation at points away from the centerline. We thus replace  $x$  by  $x + \delta x$ ,  $y$  by  $y + \delta y$ , and  $\rho$  by  $\rho + \delta \rho$  in (2.12) and examine the effects of such a perturbation.

The equations of first variation are

$$(2.13) \quad \begin{aligned} \rho y \frac{k}{\partial \eta} (\delta y) + koy_\eta \delta y + y^k y_\eta \delta \rho - \frac{\rho^* q^*}{q} \eta \frac{k}{\partial \xi} (\delta x) &= 0 \\ \rho y^k \frac{\partial}{\partial \eta} (\delta x) + x_\eta y^k \delta \rho + kx_\eta \rho \delta y + \frac{\rho^* q^*}{q} \eta^k \frac{\partial}{\partial \xi} (\delta y) &= 0 \\ 2x_\xi (\rho^{\gamma-1}-1) \frac{\partial}{\partial \xi} (\delta x) + 2y_\xi (\rho^{\gamma-1}-1) \frac{\partial}{\partial \xi} (\delta y) + (\gamma-1)\rho^{\gamma-2} (x_\xi^2 + y_\xi^2) \delta \rho &= 0 \end{aligned}$$

Equations (2.13) are systems of differential equations for unknowns  $\delta x$ ,  $\delta y$ , and  $\delta \rho$ . The coefficients in the differential equations depend upon  $x$ ,  $y$ , and  $\rho$  and are considered to be smoothly varying with  $\xi$  and  $\eta$ . As numerical errors behave like rapidly varying perturbations we view the coefficients in (2.13) as being constant in a small region and seek solutions of the form

$$(2.14) \quad \delta x = \delta x_0 e^{i\beta\xi + \alpha\eta}, \quad \delta y = \delta y_0 e^{i\beta\xi + \alpha\eta}, \quad \delta \rho = \delta \rho_0 e^{i\beta\xi + \alpha\eta}$$

where  $\delta x_0$ ,  $\delta y_0$ ,  $\delta \rho_0$ ,  $\alpha$ , and  $\beta$  are constants and  $\rho$  is real and large. Putting (2.14) into (2.13) results in

$$(2.15) \quad \begin{aligned} & (\rho y^k \alpha + k\rho y_\eta) \delta y_0 + y_\eta^k y_\eta \delta \rho_0 - \frac{\rho^* q^*}{q} n^k i\beta \rho \delta x_0 = 0 \\ & (k\rho x_\eta + \frac{\rho^* q^*}{q} n^k i\beta) \delta y_0 + x_\eta y^k \delta \rho_0 + \rho y^k \alpha \delta x_0 = 0 \\ & 2y_\xi (\rho^{\gamma-1}-1) i\beta \delta y_0 + (\gamma-1) \rho^{\gamma-2} (x_\xi^2 + y_\xi^2) \delta \rho_0 + 2x_\xi (\rho^{\gamma-1}-1) i\beta \delta x_0 = 0 \end{aligned}$$

We are thus led to two systems of linear homogeneous equations in  $\delta x_0$ ,  $\delta y_0$ , and  $\delta \rho_0$ . Assuming the perturbations are nontrivial we must have that the coefficient determinants are zero. Evaluating these determinants and setting them equal to zero we are led to

$$(2.16) \quad \begin{aligned} & -\alpha^2 \rho^\gamma (\gamma-1) y^{2k} (x_\xi^2 + y_\xi^2) + \alpha [2\rho (\rho^{\gamma-1}-1) i\beta (y_\eta y_\xi + x_\eta x_\xi)] y^{2k} \\ & - k y n y^k (\gamma-1) \rho^\gamma (x_\xi^2 + y_\xi^2) + \beta^2 \frac{\rho^* q^*}{q} n^k [\frac{\rho^* q^*}{q} n^k (\gamma-1) \rho^{\gamma-2} (x_\xi^2 + y_\xi^2) \\ & - 2y^k (\rho^{\gamma-1}-1) (x_\eta y_\xi - x_\xi y_\eta)] - ikn^k \beta \frac{\rho^* q^*}{q} \rho^{\gamma-1} (\gamma-1) (x_\xi^2 + y_\xi^2) x_\eta = 0 \end{aligned}$$

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Using (2.12) and the fact that  $\beta$  is being assumed large the above reduces to

$$\alpha^2 \rho^\gamma (\gamma-1) y^{2k} = \beta^2 \left(\frac{\rho^* q^*}{\gamma}\right)^2 n^{2k} \left[ (\gamma-1) \rho^{\gamma-2} + \frac{2}{\rho} (\rho^{\gamma-1} - 1) \right]$$

or

$$(2.17) \quad \alpha^2 = \left(\frac{\rho^* q^*}{\rho \gamma}\right)^2 \beta^2 \left(\frac{n}{y}\right)^{2k} (1-M^2)$$

Equations (2.17) indicate what we would expect. In the supersonic (hyperbolic) region,  $M > 1$ , so  $\alpha$  is purely imaginary and small perturbations will remain small. On the other hand in the subsonic (elliptic) region,  $M < 1$ , so  $\alpha$  can be positive indicating that a small perturbation once introduced would grow as  $n$  increases.

## CHAPTER III

## THE NUMERICAL WORK

We are interested here in developing a numerical scheme which could be used to determine nozzle contours in a uniform manner throughout the nozzle. Our procedure will be to introduce a finite difference method to solve the equations (2.12) as an initial value problem with some Mach number or velocity array prescribed along the centerline.

There are many ways in which one could introduce finite differences to approximate the differential equations we are considering. The one used here has been chosen for its numerical simplicity. We form a rectangular network of points  $(\xi_j, \eta_m)$  with spacings  $\Delta\xi$ ,  $\Delta\eta$  and introduce the notation  $y_{j,m} = y(\xi_j, \eta_m)$ , etc. The difference equations can then be written

$$(3.1) \quad \begin{aligned} \frac{y_{j,m+1} - \frac{y_{j+1,m} + y_{j-1,m}}{2}}{\Delta\eta} &= \frac{\rho^* q^*}{\rho_{j,m} q_j} \left( \frac{2\eta_m}{y_{j+1,m} + y_{j-1,m}} \right)^k \frac{x_{j+1,m} - x_{j-1,m}}{2\Delta\xi} \\ \frac{x_{j,m+1} - \frac{x_{j+1,m} + x_{j-1,m}}{2}}{\Delta\eta} &= - \frac{\rho^* q^*}{\rho_{j,m} q_j} \left( \frac{2\eta_m}{y_{j+1,m} + y_{j-1,m}} \right)^k \frac{y_{j+1,m} - y_{j-1,m}}{2\Delta\xi} \\ \rho_{j,m} &= \left[ 1 - \frac{\gamma-1}{2} \frac{\frac{q_j^2}{x_{j+1,m} - x_{j-1,m}}^2 + \frac{y_{j+1,m} - y_{j-1,m}}{2\Delta\xi}^2}{\left( \frac{x_{j+1,m} - x_{j-1,m}}{2\Delta\xi} \right)^2} \right]^{\frac{1}{\gamma-1}} \end{aligned}$$

To solve (2.12) using (3.1) we proceed as follows. We first set  $x_{j,0} = \xi_j$  and  $y_{j,0} = 0$ . The array  $\bar{\xi}_j$  is either directly prescribed or obtained from a given Mach number distribution using the nondimensional form of equation (2.3). With this information  $\rho_{j,0}$  can be determined and from this  $x_{j,1}$  and  $y_{j,1}$ . Using these new arrays for  $x$  and  $y$ ,  $\rho_{j,1}$  is computed and the procedure continues

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inductively. Thus, when all the information for subscript  $m$  and less is known the  $x$  and  $y$  arrays for  $m + 1$  can be computed. In the program actually written here, only every other point was calculated at each step. Thus, for example (see Figure 2) the point 10 is calculated from 1, 2, and 3, then the points 12 and 14 would also be calculated but not the points 11, 13, and 15. Similarly, going to the next step, the points 10, 11, and 12 would be used in calculating 19. The point 21 would also be computed, but not the point 20. Note that with each step in the  $\eta$  direction 1 point is lost from each side of the  $\xi$  array. More will be said about this in Section IV.

The only deviation from this procedure occurs on the initial step of the process for the axisymmetric flow case. Here it is necessary to introduce the approximation

$$\frac{y}{\eta} \approx \frac{\partial y}{\partial \eta}$$

to circumvent the difficulty arising from the indeterminacy of  $\frac{\eta}{y}$  along  $\eta = 0$ . Using this approximation, we obtain for the first step

$$(3.2) \quad \frac{\partial y}{\partial \eta} = \left( \frac{\rho^* q^*}{\rho^* q^*} \right)^{1/2}$$

(since  $\frac{\partial x}{\partial \xi} = 1$  along  $\eta = 0$ ) or in the finite difference form

$$(3.3) \quad y_{j,1} = \left( \frac{\rho_j^* q_j^*}{\rho_{j,0}^* q_{j,0}} \right)^{1/2} \Delta \eta$$

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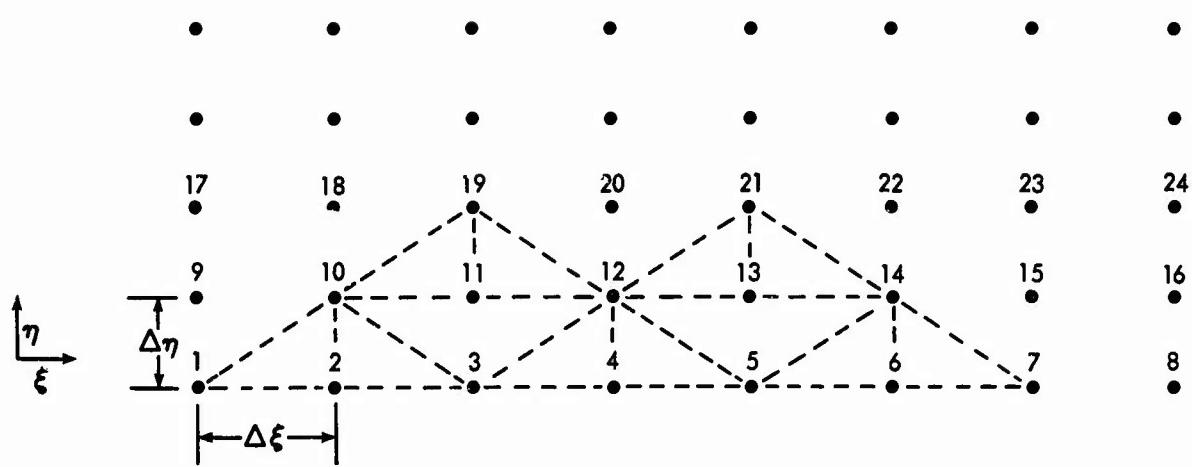


FIG. 2  $(\xi, \eta)$  MESH

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The iterative procedure outlined above is terminated when a desired stream-line,  $n_d$ , is reached. In the work described here this  $n_d$  was chosen from considerations allowing comparison with the results of others. As an example of how one might determine  $n_d$ , we consider the following case. It was desired to compute nozzles with nondimensional test section height equal to 1. From (2.4) we have

$$\psi_y = \rho y^k u$$

where  $\rho u$  is constant in the test section. Thus, integrating from the centerline,  $y = 0$ , to the contour in the test section,  $y = 1$ , one finds

$$\psi_d = \frac{\rho u}{k+1}$$

or from (2.11)

$$\rho^* q^* \frac{n_d^{k+1}}{k+1} = \frac{\rho u}{k+1}$$

Thus, it follows that

$$(3.4) \quad n_d = \left( \frac{\rho u}{\rho^* q^*} \right)^{\frac{1}{k+1}}$$

This quantity can be determined using e.g., [1] once the test section Mach number is specified.

## CHAPTER IV

## RESULTS AND CONCLUSIONS

The procedure outlined in the previous section was programmed and run on an IBM 7090. The program consisted of a main program which called upon a subprogram to generate the centerline velocity array and then carried out the integration of the equations.

For the two-dimensional equations the cases run were chosen so as to allow a comparison with the report of Baron [ 2 ]. In the examples considered there was agreement in the contour coordinates to between three and four significant figures. These results seemed most encouraging especially when we consider the crude differencing we used and the instability we know to be present.

As we attempted to move further upstream into the subsonic region, the instability did become a factor and wild oscillations along the calculated contour became apparent. This is the behavior one would expect. If we recall the perturbations we made when we considered the differential equations, they were of the form  $\delta y_0 e^{i\beta\xi} + \alpha\eta$  where, by (2.17),

$$\alpha^2 = \left( \frac{\rho^* q^*}{\rho q} \right)^2 \beta^2 \left( \frac{\eta}{y} \right)^{2k} (1 - M^2)$$

Thus, for fixed  $\eta$  we must move sufficiently upstream from the sonic line for  $\alpha$ , and consequently  $e^{\alpha\eta}$ , to become large enough to distort the calculations. We point out, however, that as we make our centerline Mach number array pass through  $M = 1$  with steeper slope, our calculations will break down closer to the sonic line.

In the axisymmetric case we compared our results in the supersonic region with those of Glowacki [ 6 ], who used the method of characteristics, and in the

transonic region with those of Hopkins and Hill [ 7 ], [ 8 ], who used a series method. For the Glowacki cases  $n_d$  was chosen (as described in the previous section) so as to provide a nondimensional test section height of 1. The coordinates obtained agreed to about three significant figures. In the transonic comparisons  $n_d$  was chosen so as to have the nondimensional throat height equal to 1. These were provided by Hopkins and Hill who determined them by iteration. Unfortunately, the  $n_d$ 's were so large here as to cause a divergent, oscillatory condition almost everywhere. In an attempt to be able to make some type of comparison a very coarse grid was employed ( $\Delta\xi, \Delta\eta \sim 0.1$  rather than  $\sim 0.01$  or  $0.001$  as in the previously discussed cases) and a disagreement in the results in the third significant figure was observed. This again was considered quite good considering how coarse the grid was.

The two primary difficulties with the method are the instability in the subsonic region and the loss of the end points at each step (see discussion in Chapter III). The former prohibits us from obtaining the entire subsonic region and always leaves us suspicious as to how accurate are the values we obtain. The latter difficulty is particularly important in the design of short nozzles. In this case we may not have many mesh points to work with and thus, by the time we reach the contour, much of the region of interest is lost. If we attempt to decrease the mesh size to provide us with additional points, we must take more steps in the  $\eta$  direction and thus are again in the same situation.

In the introduction we mentioned that an alternate method is desirable to avoid patching and for use in short nozzles. The above results indicate that such a need still exists. Short nozzles and nozzles with steep Mach number gradients do not yield themselves nicely to the method described herein. For long nozzles where the Mach number gradient is not large this method does seem to

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provide a method of getting the transonic and supersonic regions in a uniform way. Some patching must still be done, however, to get the contour further upstream in the subsonic region.

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LIST OF SYMBOLS

a - velocity of sound  
k - dimension parameter (=0 for two-dimensional; =1 for axisymmetric)  
L - normalizing length  
M - Mach number ( $=q/a$ )  
p - pressure  
 $\vec{q}, q$  - velocity vector, velocity magnitude  
 $x, y$  -  
 $u, v$  - velocity components in x and y directions, respectively  
 $\gamma$  - ration of specific heats (=1.4 for air)  
 $\eta$  - streamline parameter  
 $\xi$  - potential line parameter  
 $\rho$  - density  
 $\phi$  - potential function  
 $\psi$  - stream function  
 $-_o$  - stagnation condition  
 $-_d$  - design condition  
 $-^*$  - throat condition  
 $-$  - centerline condition

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13. ABSTRACT This paper is concerned with a method of finite differences for determining two-dimensional and axisymmetric supersonic nozzle contours. The approach taken is to specify a Mach number or velocity array along the entire centerline of the nozzle and then to integrate the equations numerically to obtain the desired nozzle shape. In spite of the fact that the original problem is not "well posed" in the subsonic region, reasonable results were found provided the Mach number gradient was not too steep in a neighborhood of the sonic line.		

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